

**MINISTRY OF EDUCATION AND TRAINING
UNIVERSITY OF TRANSPORT AND COMMUNICATIONS**

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**RESEARCH ON THE CHARACTERISTICS AND CONTROL OF
ELECTRIC ASSIST STEERING SYSTEM IN CARS**

**Major: Power Mechanical Engineering
Code No: 9520116**

ABSTRACT OF DOCTORAL DISSERTATION

HA NOI - 2026

The dissertation was completed at:
Department of Automotive Engineering - Faculty of Mechanical
Engineering
UNIVERSITY OF TRANSPORT AND COMMUNICATIONS

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Counter-argument 1:

Counter-argument 2:

The dissertation was defended before the University dissertation jury at:
UNIVERSITY OF TRANSPORT AND COMMUNICATIONS
At on,

The dissertation can be accessed at the library of UNIVERSITY OF TRANSPORT AND COMMUNICATIONS and the National Library of Vietnam.

LIST OF SYMBOLS

Ký hiệu	Đơn vị	Thông số	Ký hiệu	Đơn vị	Thông số
M_{mm}, M_{mg}	$[Nm]$	Moment of inertia of	y_r	$[m]$	Rack position
M_{md}, M_{mi}		l_5, l_6, l_7, l_8			
T_d	$[Nm]$	Driver torque	ρ	$[m]$	Turning radius
T_{di}	$[Nm]$	Transition point torque	m	$[kg]$	Vehicle mass
		Minimum driver torque			
T_{dmin}	$[Nm]$	to activate the assist motor	m_c	$[kg]$	Column mass
T_{dmax}	$[Nm]$	Maximum driver torque	m_m	$[kg]$	Motor mass
$T_{dmax}(v_i)$	$[Nm]$	Maximum driver torque at v_i	m_p	$[kg]$	Pinion mass
$T_{dmax}(0)$	$[Nm]$	Maximum driver torque at $v = 0$	m_r	$[kg]$	Rack mass
		Steering torque when assist motor is activated (nonlinear characteristic)			
T_{dat}	$[Nm]$	The torque when the assist steering is activated	m_{req}	$[kg]$	Front axle inertia converted to rack mass
T_{drq}	$[Nm]$	Transition point torque (mix characteristic)	m_f	$[kg]$	l_5, l_6, l_7, l_8 mass
T_{int}	$[Nm]$	Assist torque	m_r	$[kg]$	Front axle mass
T_a	$[Nm]$	Actual assist torque	m'	$[kg]$	Axle mass
T_{aas}	$[Nm]$	Required assist torque	m'_f	$[kg]$	Total sprung mass
T_{arq}	$[Nm]$	Maximum assist torque	m'_r	$[kg]$	Front axle mass
T_{amax}	$[Nm]$	Assist motor torque	m''	$[kg]$	Rear axle mass
T_m	$[Nm]$	Actual assist motor torque	m''_f	$[kg]$	Total unsprung mass
T_{mas}	$[Nm]$	Road reaction torque	m''_r	$[kg]$	Front axle unsprung mass
T_r	$[Nm]$	Right wheel road reaction torque	J_d	$[kgm^2]$	Rear axle unsprung mass
T_{rr}	$[Nm]$	Left wheel road reaction torque	J_m	$[kgm^2]$	Column inertia
T_{rl}	$[Nm]$	Random resistance torque	J_p	$[kgm^2]$	Motor inertia
T_{rd}	$[Nm]$	Load torque	J_5, J_6, J_7, J_8	$[kgm^2]$	Pinion inertia
T_L	$[Nm]$	Sensing torque	J_w	$[kgm^2]$	Inertia of l_5, l_6, l_7, l_8
T_s	$[Nm]$	Normal and tangential inertial resistance forces of the links l_5, l_6, l_7, l_8	J_r	$[kgm^2]$	Wheel inertia
F_{mi}^n, F_{mi}^T ($i = 5 \div 8$)	$[N]$	Normal and tangential reactions of the rack and at the joints A, B, D, E	J_l	$[kgm^2]$	Inertia of the lateral deformation zone of the right tire
F_4, N, F_A^n F_A^T, F_B^n, F_B^T F_D^n, F_D^T F_E^n, F_E^T	$[N]$		J_z	$[kgm^2]$	Inertia of the lateral deformation zone of the left tire

F_{Cx}, F_{Cy} F_{Fx}, F_{Fy}	[N]	Reaction forces in the OX and OY directions at the joints C, E	B_r	[N/m]	Vehicle moment of inertia
F_{req}	[N]	Reaction force applied to the rack	B_{req}	[N/m]	Rack viscous friction
F_{Ti}	[N]	Longitudinal force	C_{spf}	[N/m]	Equivalent viscous friction from wheel to rack
F_{ni}	[N]	Lateral force	C_{spr}	[N/m]	Front axle suspension stiffness
F_{zi}	[N]	Vertical force	K_{req}	[N/m]	Rear axle suspension stiffness
F_{Tf}	[N]	Longitudinal force at the front axle	K_5	[N/m]	Steer tire stiffness converted to rack stiffness
F_{Tr}	[N]	Longitudinal force at the rear axle	K_6	[N/m]	l_5 link of stiffness
F_{nf}	[N]	Lateral force at the front axle	K_7	[N/m]	l_6 link of stiffness
F_{zf}	[N]	Vertical (normal) force at the front axle	K_8	[N/m]	l_7 link of stiffness
F_{zr}	[N]	Vertical (normal) force at the rear axle	F_r	[N/m]	l_8 link of stiffness
P_{max}	[N]	Maximum force acting on the steering wheel	K_r	[N/m]	Rack dry friction torque
Z_f	[N]	Front axle weight distribution	C_{axf}	[Nm/ rad]	Rack stiffness
Z_r	[N]	Rear axle weight distribution	C_{axr}	[Nm/ rad]	Front axle roll angle stiffness
α_f	[Rad]	Front tire side slip angle	C_{ax}	[Nm/ rad]	Rear axle roll angle stiffness
α_{fl}	[Rad]	Left front tire side slip angle	M_c	[Nm/ rad]	Axle roll angle stiffness
α_{fr}	[Rad]	Right front tire side slip angle	M_m	[Nm/ rad]	Column dry friction torque
α_r	[Rad]	Rear tire side slip angle	M_p	[Nm/ rad]	Assist motor dry friction torque
α_i	[Rad]	The i^{th} tire side slip angle	$K_{\alpha i}$	[Nm/ rad]	Pinion dry friction torque
β	[Rad]	Vehicle side slip angle	$K_{\alpha f}$	[Nm/ rad]	Tire lateral stiffness coefficient
φ_d	[rad]	Steering column angle	$K_{\alpha r}$	[Nm/ rad]	Front tire sideslip coefficient (lateral stiffness)
φ_m	[rad]	Assist motor shaft angle	K_s	[Nm/ rad]	Rear tire sideslip coefficient (lateral stiffness)
φ_p	[rad]	Pinion angle	K_c	[Nm/	Steering system

φ_{wr}	[rad]	Right wheel angle	K_m	rad] [Nm/ rad]	stiffness Steering column stiffness
φ_{wl}	[rad]	Left wheel angle	K_p	[Nm/ rad]	Assist motor shaft stiffness
φ_5	[rad]	l_5 link of angle	v	[m/s]	Pinion stiffness
φ_6	[rad]	l_6 link of angle	$v_{\dot{\psi}}$	[m/s]	Vehicle speed
φ_7	[rad]	l_7 link of angle	v_f, v_r	[m/s]	Vehicle characteristic speed
φ_8	[rad]	l_8 link of angle	a_y	[m/s ²]	Front and rear wheel longitudinal speeds
θ_r	[rad]	Rotor angle	B_c	[(Nm)/ (rad/s)]	Lateral acceleration
ψ	[Rad]	Vehicle yaw angle	B_m	[(Nm)/ (rad/s)]	Steering column viscous friction
δ_i	[Rad]	Steer angle	B_p	[(Nm)/ (rad/s)]	Assist motor shaft viscous friction
γ_p	[Rad]	Kingpin angle	D_c	[(Nm)/ (rad/s)]	Pinion viscous friction
γ_c	[Rad]	Camber angle	D_m	[(Nm)/ (rad/s)]	Steering column damping
h	[m]	Distance between rack and front axle	D_p	[(Nm)/ (rad/s)]	Assist motor shaft damping
h''_r	[m]	Rear axle unsprung mass centre height	D_r	[(Nm)/ (rad/s)]	Pinion damping
h''_f	[m]	Front axle unsprung mass centre height	D_{af}	[(Nm)/ (rad/s)]	Rack damping
h'_f	[m]	Front axle unsprung mass centre height	D_6	[Nm/ (m/s)]	Wheel tire of damping
h'_r	[m]	Rear axle sprung mass centre height	D_7	[Nm/ (m/s)]	l_6 link of damping
h'_g	[m]	Sprung mass centre height	D_8	[Nm/ (m/s)]	l_7 link of damping
h'	[m]	Sprung mass centre to roll axis height	p	[kPa]	l_8 link of damping
L	[m]	Wheelbase	R_s	[Ω]	Tire standard pressure
l_r, l_i	[m]	Ackermann links length (i = 5 ÷ 8)	L_s	[mH]	Stator resistor
l_p	[m]	Distance between kingpins	L_{sd}, L_{sq}	[mH]	Stator coil inductance
n_k	[m]	Tire pneumatic trail	U_s	[V]	Stator coil inductance on (d – q)
n_r	[m]	Kingpin forward indicator	u_{sd}, u_{sq}	[V]	Voltage of stator
n_s	[m]	Total offset	i_{st}	[A]	Voltage of stator on (d – q)
p_f	[m]	Front axle roll centre	i_{sd}, i_{sq}	[A]	Stator current

p_r	$[m]$	height Rear axle roll centre height	φ_s	$[Wb]$	Stator current on $(d - q)$
r_b	$[m]$	Wheel radius	$\varphi_{sd}, \varphi_{sq}$	$[Wb]$	Stator magnetic linkage
r_w	$[m]$	Wheel radius	φ_f	$[Wb]$	Stator magnetic linkage on $(d - q)$
r_{sw}	$[m]$	Steering wheel radius	$\varphi_{rd}, \varphi_{rq}$	$[Wb]$	Rotor magnetic linkage
r_p	$[m]$	Pinion radius	ω_m	$[Rad/s]$	Magnetic linkage on $(d - q)$
t_f	$[m]$	Front wheel track	ω_r	$[Rad/s]$	Rotor mechanical angular velocity
t_r	$[m]$	Rear wheel track	p_n		Electrical angular velocity of the d-q reference frame
x_i	$[m]$	Ox-axis distance of the axle number i from the mass centre	$Ki(v)$		Number of motor pole-pairs
x_f	$[m]$	Ox-axis distance of the front axle from the mass centre	$K(0)$		Power assist gain coefficient
x_r	$[m]$	Oy-axis distance of the rear axle from the mass centre	$f(v, T_d)$		Power assist gain coefficient when turning in place
x'_f	$[m]$	Distance of sprung mass centre to front axle	μ		Power assist gain funtion
x'_r	$[m]$	Distance of sprung mass centre to rear axle	η		Adhesion coefficient (friction)
y_i	$[m]$	Oy-axis distance of the axle number i from the mass centre	i_s		Steering system efficiency

INTRODUCTION

The steering system plays a decisive role in a vehicle's controllability and directional stability. Electric assist steering (EPS) systems are increasingly being adopted in passenger cars, replacing traditional hydraulic assist steering systems. "Steering feel" is a crucial metric reflecting the system's suitability for the driver, directly influencing control, stability, and operational safety. Since EPS operates by directly applying torque via an electric motor, an improperly designed assist characteristic or control strategy can result in an unnatural steering feel and reduced control precision. Therefore, establishing ideal assist characteristics and appropriate control strategies, alongside evaluating steering feel based on quantitative metrics, is essential for automotive EPS systems. Despite extensive existing research, holistic approaches that tightly integrate assist characteristics, system control, and steering feel evaluation still face certain limitations. Addressing these requirements, this dissertation focuses on the topic: ***"Research on the characteristics and control of electric assist steering system in cars"*** The research findings contribute to the scientific foundation for the design, control, and evaluation of EPS systems and serve as a valuable reference for research, teaching, and practical application in the field of passenger car steering systems.

CHAPTER I: OVERVIEW OF THE RESEARCH TOPIC

1.1. OVERVIEW OF THE EPS STEERING SYSTEM

The EPS system is a power-assisted steering system that utilizes an electric motor. The C-EPS system is a type of EPS where the assist unit is located on the steering column; the assist motor provides torque to the steering column via a worm gear reduction mechanism.

1.2. STEERING FEEL

In the field of vehicle dynamics, steering feel is defined as the aggregate of feedback perceived by the driver through the steering wheel and sensory inputs—including steering torque, steering angle, and self-centering characteristics—which reflect the vehicle's motion state and road conditions. Experts in vehicle dynamics and steering system development indicate that steering feel can be characterized by a set of core attributes reflecting the interaction between the driver and the vehicle. The five key attributes are: steering torque and its variation characteristics; steering response lag; transient steering torque feel; self-centering capability; and road feedback.

1.3. DOMESTIC AND INTERNATIONAL RESEARCH

1.3.1. International research

Key research areas regarding EPS include: modeling EPS systems; control strategies; methods for generating power-assist characteristic maps; steering feel evaluation; sensor feedback processing; development of user-selectable driving modes; driver assistance features; integration with vehicle dynamics control systems; and system fault detection and diagnosis. Research areas directly relevant to this thesis include: the generation of power-assist characteristic maps; EPS control strategy and controller design; and steering feel evaluation.

Research gaps: (1) There is a lack of a theoretical basis and general structure for constructing an "ideal" power-assist characteristic map—one generated from a

model that integrates driver perception characteristics while decoupling hardware parameters from perception/driving style parameters. (2) There is no "comprehensive" EPS steering feel evaluation framework—standardized according to ISO or equivalent national standards with clear quantitative metrics—to validate the quality of power-assist characteristic maps and controllers. (3) There is no closed-loop process linking power-assist map design, control strategy development, and steering feel evaluation to verify map suitability and controller responsiveness during the product research and development phase.

1.3.2. Domestic research

In-depth research publications on EPS systems remain very limited; notable studies have primarily focused on Steer-by-Wire (SBW) systems. Regarding EPS, there is a lack of in-depth research on core aspects such as the development of power-assist characteristic maps, control strategies and methodologies, target-oriented controller design, and the systematic evaluation of steering feel.

1.3.3. Central research problem and proposed approach

This thesis aims to address the following question: *How can a comprehensive, closed-loop process be established - encompassing a theoretically grounded, generalized, and replicable method for developing power-assist characteristic maps that align with driver psychology, alongside the development of control strategies and EPS controller designs to implement these maps, while also creating a standardized, holistic framework for evaluating steering feel to validate the quality of the maps and controllers?*

1.4. RESEARCH OBJECTIVES, METHODS, AND CONTENT

1.4.1. Research objectives

Objective 1: Develop assist steering characteristic maps and ideal assist steering characteristics aligned with driver psychological patterns;

Objective 2: Research strategies and design an EPS controller to implement Objective 1;

Objective 3: Evaluate steering feel to validate Objectives 1 and 2.

1.4.2. Research scope and subject

Scope: Research on developing electric assist steering characteristics; research on developing dynamic models integrated into the assist steering characteristic map structure; research on steering feel and its evaluation criteria. Based on this, develop ideal assist steering characteristic (ASC) maps, design the EPS controller, and evaluate overall steering feel.

Subject: C-EPS (Column-type electric assist steering) system for passenger cars.

1.4.3. Research methods

Theoretical research combined with simulation and virtual testing using industrial-grade simulation software.

1.4.4. Research content

The thesis is structured into four chapters: Chapter I: Overview of the research topic; Chapter II: Development of dynamic models and controller design for the EPS system; Chapter III: Simulation-based analysis and evaluation of the EPS system; Chapter IV: Simulation-based experimental research.

CHAPTER II: DEVELOPMENT OF STEERING ASSIST CHARACTERISTIC MAPS, CONTROLLER DESIGN, EVALUATION OF EPS STEERING FEEL

The objective of Chapter 2 is to establish a comprehensive, systematic, and theoretically grounded framework for developing steering assist characteristic curves; to propose a theoretical basis for constructing ideal steering assist characteristics aligned with driver psychology across various driving styles; to design the EPS system controller; and finally, to present methods and criteria for evaluating steering feel.

2.1. DEVELOPMENT OF THE DYNAMICS MODEL

The general dynamics and EPS control research model is illustrated in Figure 2.1, wherein dynamic models are formulated for integration into the steering assist characteristic structure.

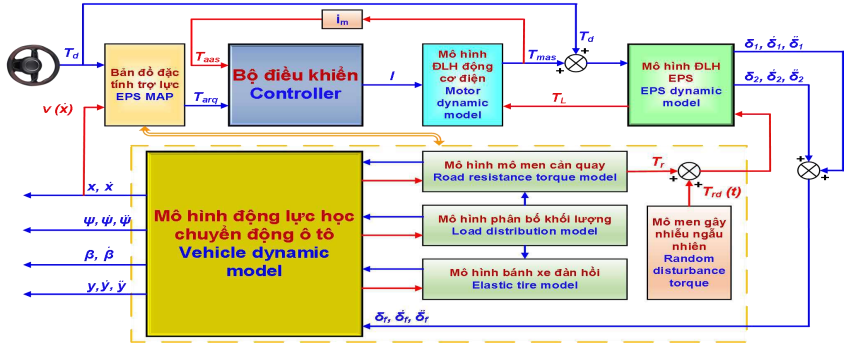


Figure 2.1. General research model for EPS dynamics and control.

2.1.1. Development of the dynamic mathematical model of the steering system

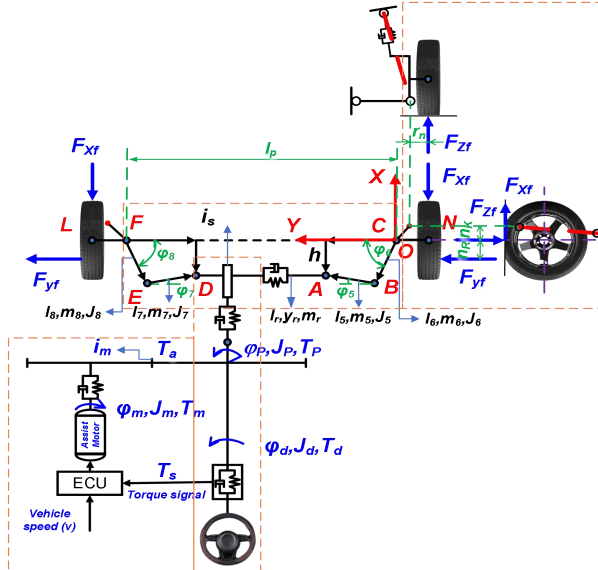


Figure 2.2. EPS dynamics model

The dynamic model of the EPS steering system is presented in Figure 2.2.

2.1.1.1. Case 1: Considering elasticity in all components

The system of differential equations representing the dynamics. The components of the steering system are as follows.

$$\left\{ \begin{array}{l} J_d \ddot{\phi}_d = T_d - [K_c(\phi_d - \phi_p) + D_c(\dot{\phi}_d - \dot{\phi}_p)] - [B_c \dot{\phi}_d + M_c \text{sign}(\dot{\phi}_d)] \\ J_m \ddot{\phi}_m = T_m - [K_m(\phi_m - i_m \phi_p) + D_m(\dot{\phi}_m - i_m \dot{\phi}_p)] - [B_m \dot{\phi}_m + M_m \text{sign}(\dot{\phi}_m)] \\ J_p \ddot{\phi}_p = i_m [K_m(\phi_m - i_m \phi_p) + D_m(\dot{\phi}_m - i_m \dot{\phi}_p)] + [K_c(\phi_d - \phi_p) + D_c(\dot{\phi}_d - \dot{\phi}_p)] \\ \quad - [K_p(\phi_p - y_r/r_p) + D_p(\dot{\phi}_p - \dot{y}_r/r_p)] - [B_p \dot{\phi}_p + M_p \text{sign}(\dot{\phi}_p)] \\ m_r \ddot{y}_r = [K_p(\phi_p - y_r/r_p) + D_p(\dot{\phi}_p - \dot{y}_r/r_p)]/r_p - [K_r(y_r - l_5 \phi_5) + \\ D_r(\dot{y}_r - l_5 \dot{\phi}_5)] - [K_r(y_r - l_7 \phi_7) + D_r(\dot{y}_r - l_7 \dot{\phi}_7)] - [B_r \dot{y}_r + F_r \text{sign}(\dot{y}_r)] \\ J_5 \ddot{\phi}_5 = [K_r(y_r - l_5 \phi_5) + D_r(\dot{y}_r - l_5 \dot{\phi}_5)]/l_5 - \\ [K_5(\phi_5 - \phi_6) + D_5(\dot{\phi}_5 - \dot{\phi}_6)] - M_5 \text{sign}(\dot{\phi}_5) \\ J_7 \ddot{\phi}_7 = [K_r(y_r - l_7 \phi_7) + D_r(\dot{y}_r - l_7 \dot{\phi}_7)]/l_7 - \\ [K_7(\phi_7 - \phi_8) + D_7(\dot{\phi}_7 - \dot{\phi}_8)] - M_7 \text{sign}(\dot{\phi}_5) \\ J_6 \ddot{\phi}_6 = [K_5(\phi_5 - \phi_6) + D_5(\dot{\phi}_5 - \dot{\phi}_6)] - \\ [K_6(\phi_6 - \phi_{wr}) + D_6(\dot{\phi}_6 - \dot{\phi}_{wr})] - M_6 \text{sign}(\dot{\phi}_6) \\ J_8 \ddot{\phi}_8 = [K_7(\phi_7 - \phi_8) + D_7(\dot{\phi}_7 - \dot{\phi}_8)] - \\ [K_8(\phi_8 - \phi_{wl}) + D_8(\dot{\phi}_8 - \dot{\phi}_{wl})] - M_8 \text{sign}(\dot{\phi}_8) \\ J_w \ddot{\phi}_{wr} = [K_6(\phi_6 - \phi_{wr}) + D_6(\dot{\phi}_6 - \dot{\phi}_{wr})] - [K_{af} \alpha_{fr} + D_{af} \dot{\alpha}_{fr}] \\ J_w \ddot{\phi}_{wl} = [K_8(\phi_8 - \phi_{wl}) + D_8(\dot{\phi}_8 - \dot{\phi}_{wl})] - [K_{af} \alpha_{fl} + D_{af} \dot{\alpha}_{fl}] \\ J_r \ddot{\alpha}_{fr} = [K_{af} \alpha_{fr} + D_{af} \dot{\alpha}_{fr}] - T_{rr} \\ J_l \ddot{\alpha}_{fl} = [K_{af} \alpha_{fl} + D_{af} \dot{\alpha}_{fl}] - T_{rl} \end{array} \right. \quad (2.1)$$

2.1.1.2. Case 2: Assuming links l_5, l_7, l_6, l_8 are absolutely rigid, while the remaining links are elastic, the equation of motion for the rack is expressed as follows:

$$m_{req} \ddot{y}_r = [K_p(\phi_p - y_r/r_p) + D_p(\dot{\phi}_p - \dot{y}_r/r_p)]/r_p - [K_{req} y_r + D_{req} \dot{y}_r] - [B_{req} \dot{y}_r + F_{req} \text{sign}(\dot{y}_r)] \quad (2.2)$$

The differential equation for Ackermann steering geometry is as follows:

$$\left\{ \begin{array}{l} J_5 \ddot{\phi}_5 = F_A^T l_5/2 + F_B^T l_5/2 \\ J_7 \ddot{\phi}_7 = F_E^T l_7/2 + F_D^T l_7/2 \\ (J_6 - m_6 l_{w1}) \ddot{\phi}_6 = l_6 F_B^n \sin(\phi_6 - \phi_5) + l_6 F_B^T \sin(\phi_6 - \phi_5) - T_{rr} \\ (J_8 - m_8 l_{w1}) \ddot{\phi}_8 = l_8 F_E^n \sin(\phi_8 - \phi_7) + l_8 F_E^T \sin(\phi_8 - \phi_7) - T_{rl} \end{array} \right. \quad (2.3)$$

Wherein the reaction forces are determined by the following system of equations:

$$\left\{ \begin{array}{l} F_D^n \sin \phi_7 + F_A^n \sin \phi_5 - F_D^T \cos \phi_7 + F_A^T \cos \phi_5 + N_r = 0 \\ F_D^n \cos \phi_7 + F_A^n \cos \phi_5 + F_D^T \sin \phi_7 + F_A^T \sin \phi_5 + m_{req} \ddot{y}_r \\ \quad = [K_{req} y_r + D_{req} \dot{y}_r] - [B_{req} \dot{y}_r + F_{req} \text{sign}(\dot{y}_r)] \\ N_r l_r/2 + F_A^T l_r \cos \phi_5 - F_A^n l_r \sin \phi_5 = 0 \\ F_B^T + m_5 l_5 \ddot{\phi}_5/2 - F_A^T = 0 \\ F_A^n - m_5 l_5 \dot{\phi}_5^2/2 - F_B^n = 0 \\ F_A^T l_5/2 + F_B^T l_5/2 - J_5 \ddot{\phi}_5 = 0 \\ F_{Cx} + F_B^n \sin \phi_5 - F_B^T \cos \phi_5 - m_6 \ddot{\phi}_6 = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l}
N_r l_r / 2 + F_A^T l_r \cos \varphi_5 - F_A^n l_r \sin \varphi_5 = 0 \\
F_B^T + m_5 l_5 \ddot{\varphi}_5 / 2 - F_A^T = 0 \\
F_A^n - m_5 l_5 \dot{\varphi}_5^2 / 2 - F_B^n = 0 \\
F_A^T l_5 / 2 + F_B^T l_5 / 2 - J_5 \ddot{\varphi}_5 = 0 \\
F_{Cx} + F_B^n \sin \varphi_5 - F_B^T \cos \varphi_5 - m_6 \ddot{\varphi}_6 = 0 \\
F_{Cy} - F_B^n \cos \varphi_5 - F_B^T \sin \varphi_5 - m_6 l_6 \ddot{\varphi}_6^2 / 2 = 0 \\
l_6 F_B^n \sin(\varphi_6 - \varphi_5) + l_6 F_B^T \sin(\varphi_6 - \varphi_5) + m_6 l_{w1} \ddot{\varphi}_6 - T_{rr} = J_6 \ddot{\varphi}_6 \quad (2.4) \\
F_E^T + m_7 l_7 \ddot{\varphi}_7 / 2 - F_D^T = 0 \\
F_E^n - m_7 l_7 \dot{\varphi}_7^2 / 2 - F_D^n = 0 \\
F_E^T l_7 / 2 + F_D^T l_7 / 2 - J_7 \ddot{\varphi}_7 = 0 \\
-F_{Fx} + F_E^n \sin \varphi_7 + F_F^T \cos \varphi_7 + m_8 \ddot{\varphi}_8 = 0 \\
-F_{Fy} - F_E^n \cos \varphi_7 - F_E^T \sin \varphi_7 + m_8 l_8 \ddot{\varphi}_8^2 / 2 = 0 \\
l_8 F_E^n \sin(\varphi_8 - \varphi_7) + l_8 F_E^T \sin(\varphi_8 - \varphi_7) + m_8 l_{w1} \ddot{\varphi}_8 - T_{rl} = J_8 \ddot{\varphi}_8
\end{array} \right.$$

2.1.2. Developing a vehicle motion dynamics model

Developing a vehicle dynamics model. A two-track vehicle dynamics model is used to calculate vehicle dynamic parameters. A single-track dynamics model is used to determine transfer functions and inverse transfer functions. The two-track dynamics model is illustrated in Figure 2.3.

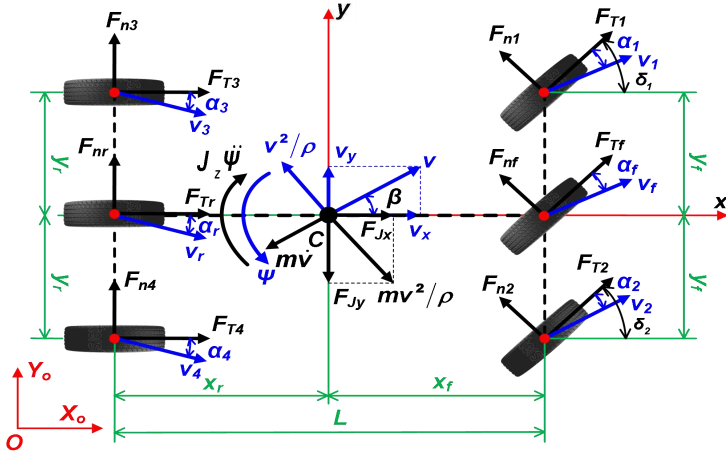


Figure 2.3. Mô hình động lực học chuyển động hai vết

System of dynamic equations governing the motion of a two-track model vehicle:

$$\left\{ \begin{array}{l}
\dot{v} = v(\dot{\psi} + \dot{\beta})\beta + \frac{1}{m} (\sum_{i=1}^4 F_{Ti} \cos \delta_i - \sum_{i=1}^4 F_{ni} \sin \delta_i) \\
\dot{\beta} = -\frac{\dot{v}}{v} \beta - \dot{\psi} + \frac{1}{m} (\sum_{i=1}^4 F_{Ti} \sin \delta_i + \sum_{i=1}^4 F_{ni} \cos \delta_i) \\
\dot{\psi} = \frac{1}{J_z} [\sum_{i=1}^4 F_{Ti} (x_i \sin \delta_i - y_i \cos \delta_i) + \sum_{i=1}^4 F_{ni} (y_i \sin \delta_i - x_i \cos \delta_i)]
\end{array} \right. \quad (2.5)$$

The single-particle motion dynamics model is shown in Figure 2.4.

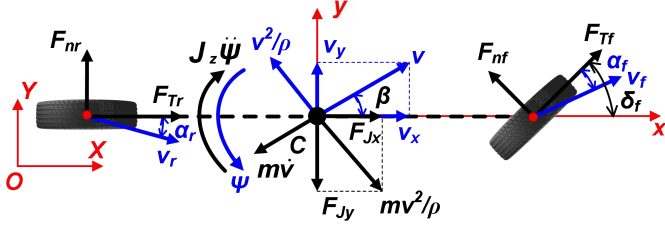


Figure 2.4. Single-track vehicle dynamics model

he differential equation for the single-track vehicle dynamics model is as follows:

$$\begin{cases} mv\dot{\beta} + (K'_{af} + K'_{ar})\beta + [mv^2 - (K'_{ar}x_r - K'_{af}x_f)]\frac{\dot{\psi}}{v} = K'_{af}\frac{\varphi_d}{i_s} \\ J_z\dot{\psi} + (K'_{af}x_f^2 + K'_{ar}x_r^2)\frac{\dot{\psi}}{v} - (K'_{ar}x_r - K'_{af}x_f)\beta = K'_{af}x_f\frac{\varphi_d}{i_s} \end{cases} \quad (2.6)$$

2.1.3. Developing a dynamic model of an elastic wheel.

The reaction forces from the road surface acting on the wheel are shown in Figure 2.5. Where v_{xi} , v_{yi} , v_i represent the longitudinal, lateral, and resultant velocities of the wheel, respectively; β_i is the angle between the velocity vector v_i and the direction of the vehicle body's motion; F_{zi} is the vertical reaction force; F_{ni} is the lateral reaction force; F_{Ti} is the longitudinal reaction force.

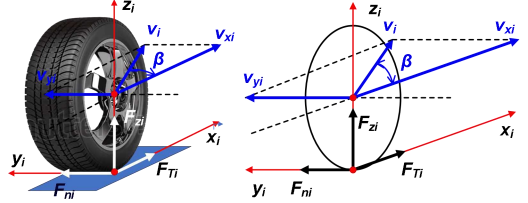


Figure 2.5. Forces exerted by the road on the wheels

Lateral slip angle of the wheels: $\alpha_i = \beta_i - \delta_i$ (2.7)

Slip angle β_i determined by: $\beta_i = \arctan\left(\frac{v_{yi}}{v_{xi}}\right)$ (2.8)

2.1.4. Lateral reaction force exerted by the road surface on the wheels

The lateral reaction force exerted by the road surface on the wheels F_{nj} is determined using Pacejka's empirical "Magic Formula."

$$F_n(\alpha) = D \sin(\text{Carctan}(B\alpha - E(B\alpha - \arctan(B\alpha)))) \quad (2.9)$$

Where: C is shape factor; $D = a_1 F_z^2 + a_1 F_z$ is peak factor of F_n ; $B = \frac{a_3 \sin(\arctan(a_4 F_z^2 + a_5 F_z))}{CD}$ is stiffness factor; $\widehat{BCD} = a_3 \sin(\arctan(a_4 F_z^2 + a_5 F_z))$ is the tire stiffness; $E = a_6 F_z^2 + a_7 F_z + a_8$ is curvature factor; $a = [a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8]$ are coefficients depending on the tire type and model.

2.1.5. Developing a model to determine the load distributed on the wheels

$$\begin{cases} F_{z1} = \frac{F_{zf}}{2} - \Delta Z_f \\ F_{z2} = \frac{F_{zf}}{2} + \Delta Z_f \\ F_{z3} = \frac{F_{zr}}{2} - \Delta Z_r \\ F_{z4} = \frac{F_{zr}}{2} + \Delta Z_r \end{cases} \quad (2.10)$$

The model for determining the load distribution, Figure 2.6.

$$\Delta Z_f = m' \frac{v^2}{\rho} \frac{1}{t_f} \left(\frac{b'}{L} p_f + \frac{C_{spf}}{C_{spf} + C_{spr} - m' gh} h' \right) + \frac{m''_f}{m''} h''_f \quad (2.11)$$

$$\Delta Z_r = m' \frac{v^2}{\rho} \frac{1}{t_r} \left(\frac{a'}{L} p_r + \frac{C_{spr}}{C_{spf} + C_{spr} - m' gh} h' + \frac{m''_r}{m''} h''_r \right) \quad (2.12)$$

Where: F_{zi} - weight distribution across the wheels.

$$T_m = p_n(\varphi_{rd}i_{sq} - \varphi_{rq}i_{sd}) = p_n[\varphi_f i_{sq} + (L_{sd} - L_{sq})i_{sd}i_{sq}] \quad (2.16)$$

The equation of motion for the motor is: $J_m \dot{\omega}_m = T_m - T_L - B_m \omega_m - T_f$ (2.17)

2.1.8. Random resisting torque

2.1.8.1. White noise

$$\begin{cases} X(t) \sim \mathcal{N}(0, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x-\mu^2}{2\sigma^2}} \\ T_{rd}(t) = X(t) * h(t) \end{cases} \quad (2.18)$$

2.1.8.2. Random number

$$x(t) = R(f, \mu, \sigma, \text{Dist}) \quad (2.19)$$

2.2. DEVELOPING THE STEERING ASSIST CHARACTERISTIC MAP FOR EPS

Steering assist characteristics represent the relationship between assist torque and steering torque. A steering assist characteristic map is a graph displaying a family of steering assist characteristic curves plotted against vehicle speed. Based on the requirements for electric assist steering, the thesis establishes formulas to construct four types of steering assist maps, as presented below.

2.2.1. Linear assist steering characteristics

$$T_a = \begin{cases} 0 & \text{When } |T_d| < T_{dmin} \\ K(v)(T_d - T_{dmin}) & \text{When } T_{dmin} \leq |T_d| \leq T_{dmax} \\ T_{amax} = K(v)(T_{dmax} - T_{dmin}) & \text{When } |T_d| \geq T_{dmax} \end{cases} \quad (2.20)$$

$$\text{Vói: } K(v) = (T_\Sigma - T_d) / (T_{dmax} - T_{dmin}) \quad (2.21)$$

2.2.2. Multi-linear assist steering characteristics

$$T_a = \begin{cases} 0 & \text{When } |T_d| < T_{dmin} \\ K_0(v)(T_d - T_{dmin}) & \text{When } T_{dmin} \leq |T_d| \leq T_{d1} \\ K_1(v)(T_d - T_{d1}) + K_0(v)(T_d - T_{dmin}) & \text{When } T_{d1} \leq |T_d| \leq T_{d2} \\ \dots \\ \sum_{i=0}^m K_i(v)(T_d - T_{di}) & \text{When } T_{d(i-1)} \leq |T_d| \leq T_{di} \\ T_{amax} = K(v)(T_{dmax} - T_{d0}) & \text{When } |T_d| \geq T_{dmax} \end{cases} \quad (2.22)$$

2.2.3. Non-linear assist steering characteristics

$$T_a = \begin{cases} 0 & \text{When } |T_d| < T_{dmin} \\ f(v, T_d)f(T_d) & \text{When } T_{dmin} \leq |T_d| \leq T_{dmax} \\ T_{amax} = K(v)(T_{dmax} - T_{drq}) & \text{When } |T_d| \geq T_{dmax} \end{cases} \quad (2.23)$$

2.2.4. Linear and non-linear assist steering characteristics

$$T_a = \begin{cases} 0 & \text{When } |T_d| < T_{dmin} \\ K1(v)(T_d - T_{dmin}) & \text{When } T_{dmin} \leq |T_d| \leq T_{int} \\ f(v, T_d)f(T_d) & \text{When } T_{int} \leq |T_d| \leq T_{dmax} \\ T_{amax}^* = K(v)T_{dmax}^* & \text{When } |T_d| \geq T_{dmax} \end{cases} \quad (2.24)$$

Where: $T_{dmax}^* = T_{dmax} - T_{int}$; T_{int} - torque transition point.

2.3. DEVELOPING IDEAL ASSIST STEERING CHARACTERISTICS FOR EPS

In this thesis, ideal ASC are defined as those that provide a steering feel aligned with the psychological characteristics of the vehicle operator.

2.3.1. Theoretical basis for establishing ideal ASC

In 1953, Stevens introduced Stevens's power law, stating that sensory perception

is proportional to a power of the physical stimulus, thereby establishing a power-law relationship between the magnitude of sensation and the stimulus: $S = aI^b$ (2.25)

Where: S - sensation intensity; I - stimulus intensity; a - intensity coefficient; b - power exponent determined by the type and intensity of the physical stimulus. The thesis proposes an ideal model of the relationship between steering torque and steering movement intensity based on Stevens' power law, expressed by the following formula:

$$\begin{cases} a_y^* = \frac{b}{1+cv^k} |T_d|^n \text{sign}(T_d) = K_a |T_d|^n, \text{ Where } K_a = \frac{b}{1+cv^k} \text{sign}(T_d) \\ \varphi_d^* = \frac{b_1}{1+cv^k} |T_d|^{n_1} \text{sign}(T_d) = K_\varphi |T_d|^{n_1}, \text{ Where } K_\varphi = \frac{b_1}{1+cv^k} \text{sign}(T_d) \end{cases} \quad (2.26)$$

Where: φ_d^* , a_y^* are the steering angle and lateral acceleration desired by the driver; b, b_1 are the intensity coefficients for the steering angle and lateral acceleration; c, k represent the influence of velocity on the intensity coefficients; n, n_1 are the exponents for the steering angle and lateral acceleration.

2.3.2. Mô hình xây dựng đặc tính lái lý tưởng

The proposed analysis model for EPS characteristics, based on the "driving style-vehicle inverse dynamics-steering system inverse dynamics" framework, as Figure 2.8.

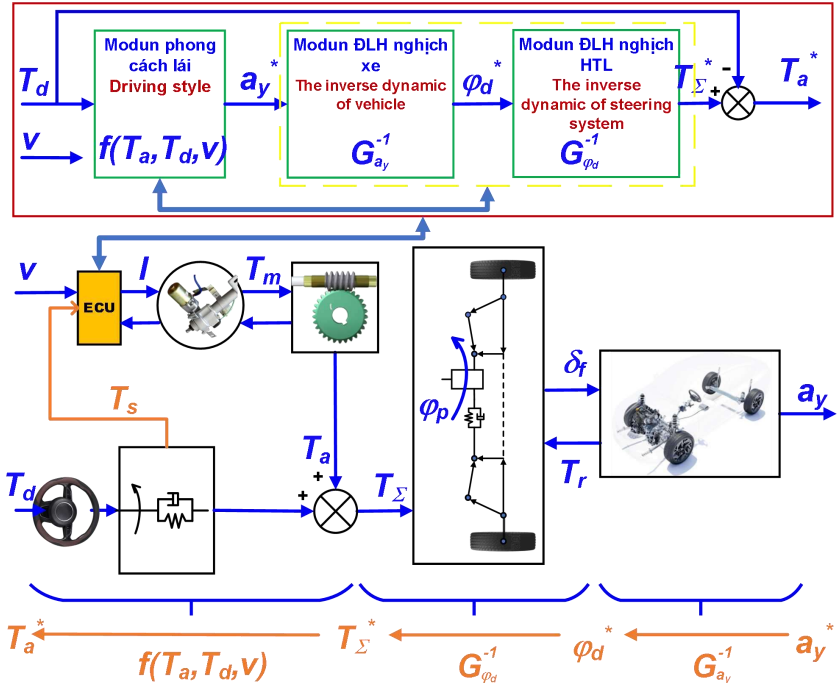


Figure 2.8. Driving-style-based EPS inverse dynamics analysis model

Based on the "Driving Style" module, the desired steering intensity expressed as the desired lateral acceleration (a_y^*) is determined in response to the input steering torque (T_d). Next, the "inverse vehicle dynamics" module calculates the desired steering angle (φ_d^*) based on the input (a_y^*). From there, the "inverse steering system

dynamics" module calculates the required total torque (T_s^*) based on the input (φ_d^*). Using (T_s^*) and (T_d), the required assist torque (T_a^*) is calculated. Based on (T_a^*), the ECU sends a control current signal (I) to the assist motor; the motor generates an assist torque (T_m) which, via the worm-and-worm-gear transmission, produces the actual assist torque ($T_a = T_a^*$). The resulting total torque ($T_s = T_a + T_d = T_s^*$) is transmitted to the pinion, causing it to rotate to the desired angle ($\varphi_p = \varphi_p^*$); subsequently, through the Ackermann linkage, the front wheels are turned to the desired angle (δ_f), thereby achieving the desired steering intensity, specifically ($a_y = a_y^*$).

2.3.3. Development of the inverse transfer function

$$\text{Inverse transfer function } a_y - \varphi_d: G_{a_y}^{-1} = \frac{\varphi_d}{a_y} \bigg|_s = i_s L \frac{1 + (v/v_\psi)^2}{v^2} \quad (2.27)$$

$$\text{Inverse transfer function } \varphi_d - T_s: G_{\varphi_d}^{-1} = \frac{\varphi_d}{T_s} \bigg|_s = \frac{i_s^2 L^2}{m x_r n_s} \frac{1 + (v/v_\psi)^2}{v^2} \quad (2.28)$$

$$(T_a - T_d - a_y) \text{ relationship according to Stevens' power law: } K_T = \frac{m x_r n_s}{i_s L} \quad (2.29)$$

$$T_a = (K_T K_a |T_d|^{n-1} - 1) T_d; \quad a_y = \frac{b}{1 + c v^k} |T_d|^n \text{sign}(T_d) = K_a |T_d|^n \quad (2.30)$$

This serves as a crucial basis for developing an ideal steering characteristic map.

2.3.4. Developing ASC based on Stevens' power law

Based on the relationship ($T_a - T_d - a_y$) governed by Stevens' power law (2.89), the thesis establishes a method for constructing ideal ASC maps, specifically for nonlinear and mixed characteristics using Stevens' power law.

2.3.4.1. Non-linear assist steering characteristic based on Stevens' power law

$$T_a = \begin{cases} 0 & \text{When } |T_d| < T_{dat} \\ (K_T K_a |T_d|^n - 1) T_d & \text{When } T_{dmax} \geq |T_d| \geq T_{dat} \\ T_{amax} = K(v) T_{dmax}^* & \text{When } |T_d^* T_d| > K(v) T_{dmax}^* \end{cases} \quad (2.31)$$

Where: T_{dat} - Minimum steering torque required to activate the power assist motor.

$$T_d^* = T_d \pm T_{dmin}; T_d^{**} = K_T K_a |T_d|^n - 1; T_{dmax}^* = T_{dmax} \pm T_{dmin} \quad (2.32)$$

2.3.4.2. Mixed assist steering assistance characteristics based on Stevens' power law

$$T_a = \begin{cases} 0 & \text{When } |T_d| < T_{dmin} \\ K(v) T_d^* & \text{When } |T_d| \geq T_{dmin} \text{ and } |T_d^{***} T_d^*| < |K(v) T_d^*| \\ T_d^{***} T_d^* & \text{When } K_1(v) |T_{dmax}^*| \geq |T_d^{***} T_d^*| \geq |K(v) T_d^*| \\ K_1(v) T_{dmax}^* & \text{When } |T_d^{***} T_d^*| > T_{amax} = K_1(v) T_{dmax}^* \end{cases} \quad (2.33)$$

Where: $T_d^{***} = K_T K_a |T_d|^n - 1$; $T_d^* = T_d \pm T_{dmin}$; $T_{dmax}^* = T_{dmax} \pm T_{int}$; T_{int} - the intersection of the line $T_a = K(v) T_d^*$ and the nonlinear curve $T_a = T_d^{***} T_d^*$.

2.3.5. Developing assist steering characteristics based on "Driving style"

2.3.5.1. Driving style

The thesis presents a method for developing assist steering characteristics based on three distinct driving styles "Comfort," "Sport," and "Multi-level" using the same vehicle configuration. These styles are applied to two types of assist steering characteristics: "non-linear" and "mixed"

2.3.5.2. Multi-level driving characteristics

The objective was to achieve: a comfortable driving feel at low speeds, a sporty feel at medium speeds, and a solid feel at high speeds. The results are as follows:

a. Non-linear assist steering characteristic

$$T_a = \left\{ \begin{array}{ll} \text{When } v \leq 20 \text{ km/h} & \\ \begin{array}{l} 0 \\ T_d^{**} T_d^* \\ K(v) T_{dmax}^* \end{array} & \begin{array}{l} \text{When } |T_d| < T_{dmin} = \sqrt[n]{1/K_T K_a} \\ \text{When } T_{dmax} \geq |T_d| \geq \sqrt[n]{1/K_T K_a} \\ \text{When } |T_d^{**} T_d^*| > K(v) T_{dmax}^* \end{array} \\ \\ \text{When } 20 \text{ km/h} < v \leq 100 \text{ km/h} & \\ \begin{array}{l} 0 \\ T_d^{****} T_d^* \\ K(v) T_{dmax}^* \end{array} & \begin{array}{l} \text{When } |T_d| < T_{dmin} = \sqrt[n-1]{1/K_T K_a} \\ \text{When } T_{dmax} \geq |T_d| \geq \sqrt[n-1]{1/K_T K_a} \\ \text{When } |T_d^{****} T_d^*| > K(v) T_{dmax}^* \end{array} \\ \\ \text{When } 100 \text{ km/h} < v < 150 \text{ km/h} & \\ \begin{array}{l} 0 \\ K2(v) (T_d \pm T_{drq}) \\ K2(v) (T_{dmax}^* \pm T_{drq}) \end{array} & \begin{array}{l} \text{When } |T_d| \leq T_{drq} \\ \text{When } |K2(v) (T_d \pm T_{drq})| \leq K2(v) |T_{dmax}^* \pm T_{drq}| \\ \text{When } |K2(v) (T_d \pm T_{drq})| > K2(v) |T_{dmax}^* \pm T_{drq}| \end{array} \\ \\ \text{When } v \geq 150 \text{ km/h} & \\ \begin{array}{l} 0 \\ K2(v) (T_d \pm T_{drq}) \\ T_{alm} \end{array} & \begin{array}{l} \text{When } T_d \leq T_{drq} \\ \text{When } |T_d| > T_{drq} \text{ v\`a } |K2(v) (T_d \pm T_{drq})| \leq T_{alm} \\ \text{When } |K2(v) (T_d \pm T_{drq})| > T_{alm} \end{array} \end{array} \right. \quad (2.34)$$

(When: $K2(v) = g_0(v_{rq}/v) + g_1v + g_2v^2 + g_3v^3 \geq 0$)
(When: $K2(v) = g_0(v_{rq}/v) + g_1v + g_2v^2 + g_3v^3 < 0$)

b. Mixed assist steering assistance characteristics

$$T_a = \left\{ \begin{array}{ll} \text{When } v \leq 20 \text{ km/h} & \\ \begin{array}{l} 0 \\ K(v) T_d^* \\ T_d^{**} \\ \pm K1(v) T_{dmax}^* \end{array} & \begin{array}{l} \text{When } |T_d| < T_{dmin} \\ \text{When } |T_d| \geq T_{dmin} \text{ v\`a } |T_d^{**}| < |K(v) T_d^*| \\ \text{When } |K1(v) T_{dmax}^*| \geq |T_d^{**}| \geq |K(v) T_d^*| \\ \text{When } |T_d^{**}| > |K1(v) T_{dmax}^*| \end{array} \\ \\ \text{When } 20 \text{ km/h} < v \leq 100 \text{ km/h} & \\ \begin{array}{l} 0 \\ K(v) T_d^* \\ T_d^{****} T_d^* \\ \pm K1(v) T_{dmax}^* \end{array} & \begin{array}{l} \text{When } |T_d| < [(K_T K_a)^{1-n} \mp T_{dmin}] \\ \text{When } |T_d| \geq [(K_T K_a)^{1-n} \mp T_{dmin}] \text{ v\`a } |T_d^{****} T_d^*| < |K(v) T_d^*| \\ \text{When } K1(v) |T_{dmax}^*| \geq |T_d^{****} T_d^*| \geq K(v) T_d^* \\ \text{When } |T_d^{****} T_d^*| > |K1(v) T_{dmax}^*| \end{array} \\ \\ \text{When } 100 \text{ km/h} < v < 150 \text{ km/h} & \\ \begin{array}{l} 0 \\ K2(v) (T_d \mp T_{drq}) \\ \pm K1(v) (T_{dmax}^* \mp T_{drq}) \end{array} & \begin{array}{l} \text{When } |T_d| \leq T_{drq} \\ \text{When } |T_d| > T_{drq} \text{ and } |K1(v) (T_{dmax}^* \mp T_{drq})| \geq |K2(v) (T_d \mp T_{drq})| \\ \text{When } |K2(v) (T_d \mp T_{drq})| > |K1(v) (T_{dmax}^* \mp T_{drq})| \end{array} \\ \\ \text{When } v \geq 150 \text{ km/h} & \\ \begin{array}{l} 0 \\ K2(v) (T_d \pm T_{drq}) \\ \mp T_{alm} \end{array} & \begin{array}{l} \text{When } |T_d| \leq T_{drq} \\ \text{When } |T_d| > T_{drq} \text{ and } T_{alm} \geq |K2(v) (T_d \mp T_{drq})| \\ \text{When } |K2(v) (T_d \mp T_{drq})| \geq T_{alm} \end{array} \end{array} \right. \quad (2.35)$$

(When: $K2(v) = g_0(v_{rq}/v) + g_1v + g_2v^2 + g_3v^3 \geq 0$)
(When: $K2(v) = g_0(v_{rq}/v) + g_1v + g_2v^2 + g_3v^3 < 0$)

2.3.5.3. Comfortable driving style

The goal is to achieve a smooth and comfortable driving feel.

a. Non-linear assist steering characteristic

$$T_d^* = T_d \pm T_{dmin}; T_{dmax} = T_{dmax} - T_{dmin}; T_d^{**} = K_T K_a |T_d^*|^n \quad (2.36)$$

$$T_a = \begin{cases} 0 & \text{When } |T_d| < T_{dmin} = \sqrt[n]{1/K_T K_a} \\ T_d^{**} T_d^* & \text{When } T_{dmax} \geq |T_d| \geq \sqrt[n]{1/K_T K_a} \\ \pm K(v) T_{dmax}^* & \text{When } |T_d^{**} T_d^*| > |K(v) T_{dmax}^*| \end{cases} \quad (2.37)$$

b. Mixed assist steering assistance characteristics

$$T_a = \begin{cases} 0 & \text{When } |T_d| < T_{dmin} \\ K(v) T_d^* & \text{When } |T_d| \geq T_{dmin} \text{ và } |T_d^{**}| < |K(v) T_d^*| \\ T_d^{**} & \text{When } |K1(v) T_{dmax}^*| \geq |T_d^{**}| \geq |K(v) T_d^*| \\ \pm K1(v) T_{dmax}^* & \text{When } |T_d^{**}| \geq |K1(v) T_{dmax}^*| \end{cases} \quad (2.38)$$

(" + " when $T_d \geq 0$, " - " when $T_d < 0$)

Trong đó:

$$\begin{cases} T_d^* = T_d \mp T_{dmin} & \text{(" + " when } T_d < 0, \text{ " - " When } T_d \geq 0) \\ T_{dmax}^* = T_{dmax} - T_{dmin} \\ T_d^{**} = K_T K_a |T_d^*|^n \end{cases} \quad (2.39)$$

2.3.5.4. Sporty driving style

The goal was to achieve a sporty driving feel. The results obtained are as follows:

a. Non-linear assist steering characteristic

$$T_a = \begin{cases} 0 & \text{Khi } |T_d| < T_{dmin} = \sqrt[n-1]{1/K_T K_a} \\ T_d^{***} T_d^* & \text{Khi } T_{dmax} \geq |T_d| \geq \sqrt[n-1]{1/K_T K_a} \\ \pm K(v) T_{dmax}^* & \text{Khi } |T_d^{***} T_d^*| > |K(v) T_{dmax}^*| \end{cases} \quad (2.40)$$

$$\text{Trong đó: } T_d^{***} = K_T K_a |T_d^*|^{n-1} - 1 \quad (2.41)$$

b. Mixed assist steering assistance characteristics

$$T_a = \begin{cases} 0 & \text{Khi } |T_d| < [(K_T K_a)^{1-n} \mp T_{dmin}] \\ K(v) T_d^* & \text{Khi } |T_d| \geq [(K_T K_a)^{1-n} \mp T_{dmin}] \text{ và } |T_d^{***} T_d^*| < |K(v) T_d^*| \\ T_d^{***} T_d^* & \text{Khi } |K1(v) T_{dmax}^*| \geq |T_d^{***} T_d^*| \geq |K(v) T_d^*| \\ \pm K1(v) T_{dmax}^* & \text{Khi } |T_d^{***} T_d^*| > |K1(v) T_{dmax}^*| \end{cases} \quad (2.42)$$

2.4. CONTROLLER DESIGN FOR EPS STEERING SYSTEM

The PID-FOC control model for the EPS is shown in Figure 2.9.

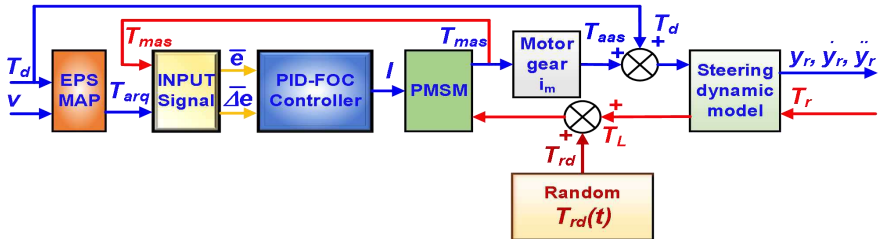


Figure 2.9. EPS control strategy diagram

The PID-FOC controller comprises several key functional modules: the "EPS MAP" module calculates the required assist torque (T_{arq}) based on the steering torque

signal (T_d) and velocity (v). The "INPUT SIGNAL" module receives the (T_{arq}) value and compares it with the actual assist torque (T_{mas}) generated by the PMSM steering motor, as obtained from the electric motor torque sensor. Based on this comparison, the module calculates the control current error (e) and the derivative of the current error (Δe), normalizing both quantities before feeding them into the "PID-FOC" control module. The "PID-FOC" module receives the normalized signal pair ($\bar{e}, \bar{\Delta e}$) and regulates the current (I) supplied to the assist motor to generate (T_{mas}). The "Steering dynamic model" module simulates the steering system dynamics using input signals comprising the assist motor torque transmitted through the reduction gear (T_{aas}), the (T_d), and the road resistance torque (T_r). This generates state feedback regarding the displacement, velocity, and acceleration of the steering rack ($y_r, \dot{y}_r, \ddot{y}_r$). Additionally, the "Random" module represents the random disturbance torque component (T_{rd}), reflecting unpredictable, stochastic inputs from the road surface.

2.5. METHODS AND CRITERIA FOR EVALUATING STEERING FEEL

2.5.1. Steering feel and the "Driver-Vehicle" closed-loop control model

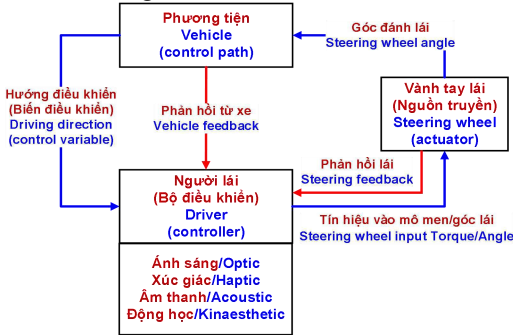


Figure 2.10. "Driver-Vehicle" closed-loop control model

The characteristics of driving feel require a simultaneous analysis of both the vehicle and the driver, forming a closed-loop "Driver-Vehicle" control system as Figure 2.10. In this system, the driver acts as the controller, detecting deviations in control variables from the desired state and adjusting actuation variables and steering angle accordingly. The driver relies on various sources of information to perform the driving task.

2.5.2. Methods for evaluating steering feel

2.5.2.1. Subjective evaluation method (SEM)

SEM conducted by drivers or test experts who possess extensive knowledge regarding desired driving characteristics—such as smoothness, comfort, or sportiness. These drivers, often drawn from diverse driving groups, provide comprehensive details on the relationship between steering inputs and vehicle responses.

2.5.2.2. Objective evaluation method

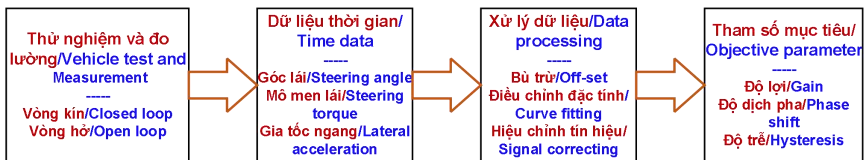


Figure 2.11. Objective testing

Objective methods are used to quantify vehicle handling characteristics, complementing subjective assessments. These involve standardized tests, such as those

defined by international ISO standards or Chinese GB/T standards. The objective testing process is illustrated in Figure 2.11.

2.5.2.3. Virtual simulation method (VSM)

The VSM utilizes advanced simulation models to evaluate vehicle dynamics and handling characteristics from the very beginning of the development process.

2.5.3. Steering feel evaluation metrics

Objective steering feel metrics are determined based on test data from characteristic curves describing the dynamic responses of the steering system and the vehicle. Key metrics include: steering stiffness, steering torque gradient, A sensitivity, vehicle body yaw sensitivity (relative to steering angle and steering torque), friction torque, hysteresis loop area, steering angle dead zone, and time lag of body yaw response (relative to steering angle and steering torque), among others. Different test procedures involve different evaluation metrics.

2.6. CHAPTER CONCLUSION

Chapter 2 has accomplished the following: It proposed a model for constructing ideal ASC, wherein the assist steering characteristic map is generated from a structured mathematical model that clearly links vehicle dynamics, the steering system, and steering feel objectives. This was achieved by decomposing the relationship between steering torque and vehicle dynamic response into three functional modules: "Driving style," "Inverse vehicle dynamics," and "Inverse steering system dynamics." Formulas were formulated to construct four types of assist steering characteristics: linear, multi-linear, non-linear, and mixed. Formulas were established to construct ideal ASC maps based on Stevens' power law for non-linear and mixed characteristics, with each type encompassing three driving styles: "Comfort," "Sport," and "Multi-level." An integrated PID-FOC controller was designed for the EPS actuator. Finally, the study systematized the concept of driving feel within a closed-loop "driver-vehicle" model and established a methodology and set of objective evaluation metrics based on international ISO and Chinese GB/T standards to assess the quality of the ASC map and the control system.

CHAPTER III: SIMULATION AND EVALUATION OF THE EPS

The objective of Chapter 3 is to apply the theoretical framework established in Chapter 2 to conduct simulations and evaluations on a sample vehicle. This process aims to validate the methodology and refine the theoretical model, serving as a basis for model reuse and scalability in research regarding ideal ASC and controller design for car EPS. A standard E-segment vehicle from the CarSim library was selected as the sample vehicle to ensure the accuracy and reliability of physical parameters.

3.1. DEVELOPMENT OF THE SIMULATION MODEL

The overall "EPS DYNAMICS & CONTROLLER" simulation model is divided into two main functional blocks: the "CONTROLLER BLOCK" and the "VEHICLE DYNAMICS BLOCK." The simulation model is illustrated in Figure 4.1.

3.2. SIMULATION AND EVALUATION OF DYNAMICS MODEL RESULTS

The dynamic simulation results for the assemblies are presented in Figures 3.1 through 3.4. These results showing variations in wheel load distribution, elastic wheel dynamics, and steady-state cornering resistance torque, align with established physical

principles, thereby validating the developed mathematical models. These models provide a reliable foundation for the design of the EPS controller.

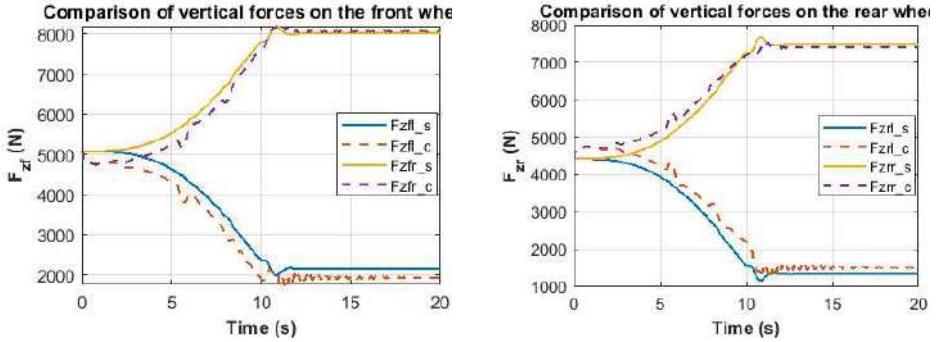


Figure 3.1. Load distribution simulation result

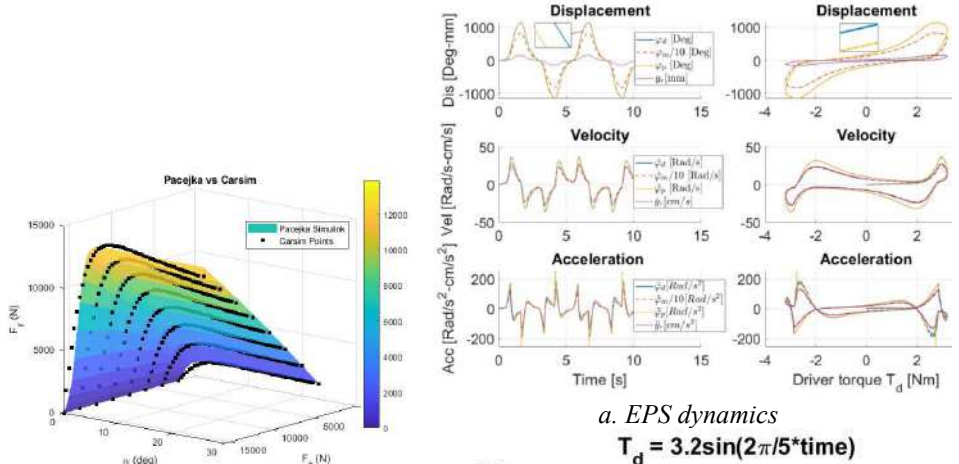


Figure 3.2. Simulation result of elastic wheel dynamics

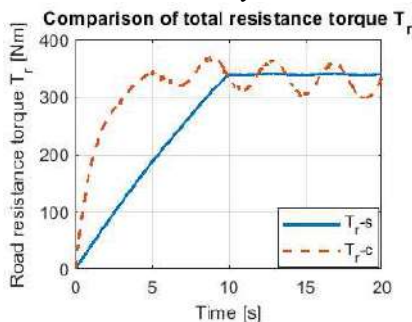
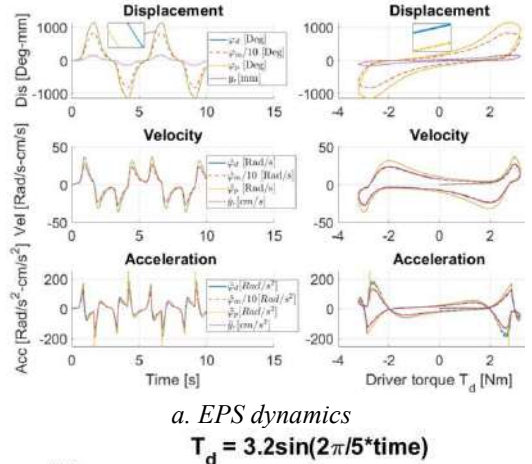
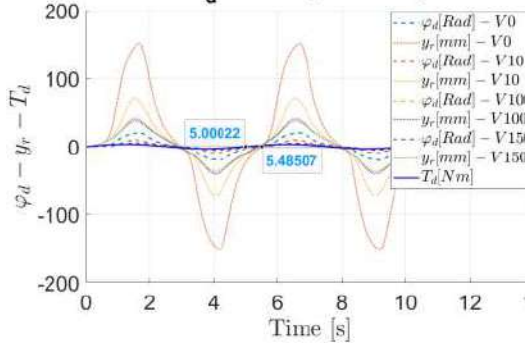


Figure 3.3. Simulation result for steering resistance torque



a. EPS dynamics

$$T_d = 3.2 \sin(2\pi/5 \cdot \text{time})$$



b. Time lag

Figure 3.4. EPS dynamics simulation results
 $T_d = 3.2 \sin(\frac{2\pi}{5} t) \text{ Nm}; (v = 0 \text{ km/h})$

3.3. SIMULATION AND EVALUATION OF EPS ASC

The simulation results for the four types of ASC are shown in Figures 3.5-3.8.

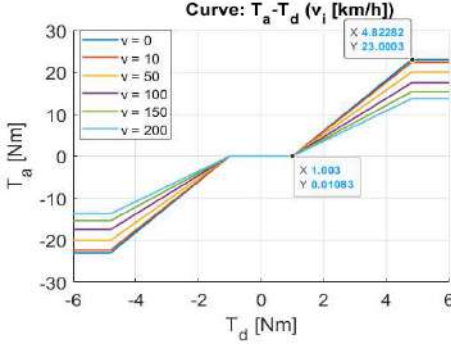


Figure 3.5. Linear characteristics

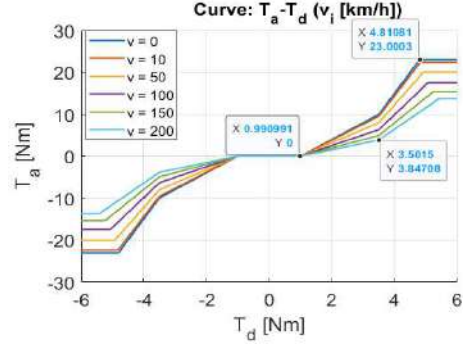


Figure 3.6. Multi-Linear characteristics

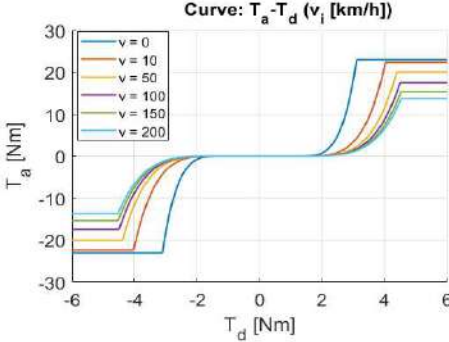


Figure 3.7. Nonlinear characteristics

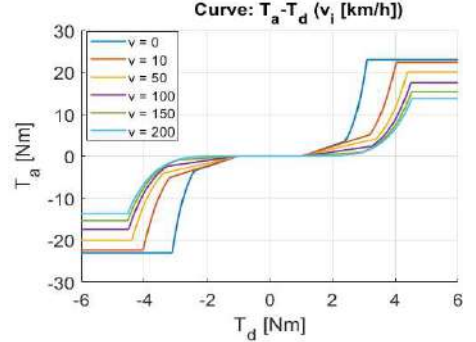
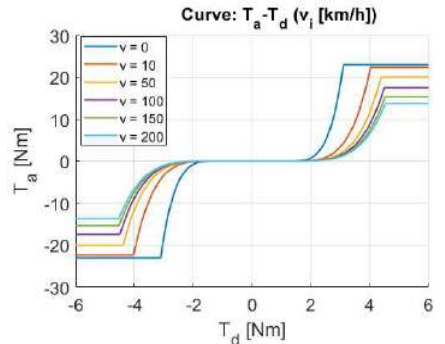


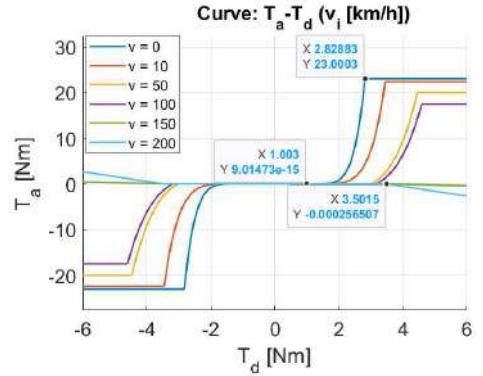
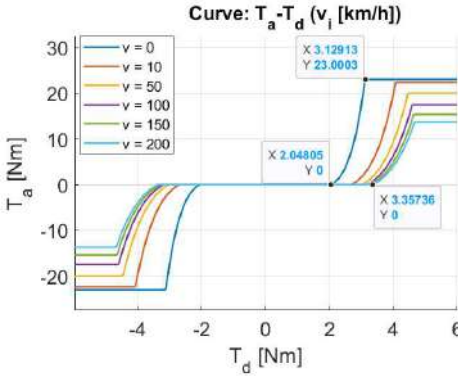
Figure 3.8. Mixed characteristics

Linear and multi-linear characteristics essentially meet assist requirements only at low speeds and struggle to satisfy the full range of stringent steering assistance demands. Non-linear and mixed characteristics, however, meet most requirements for steering assistance performance.

The simulation covers three driving styles characterized by the nonlinear profiles as Figure 3.9. The objective of simulating these distinct styles is to quantify and clarify how the strategy for distributing assist torque, based on steering torque, vehicle speed, influences steering feel, controllability, and alignment with user needs. Ultimately, this aims to establish a scientific basis for configuring various driving modes, enabling drivers to actively select the style that best suits their preferences and operating conditions.



a. Comfortable driving style



b. Sporty driving style

c. Multi-level driving style

Figure 3.9. Nonlinear characteristics with different driving styles

The resulting parameter sets for constructing ASC maps covering 04 types of assistance characteristics and 03 driving styles for non-linear and mixed characteristics:

1. Linear type:

$$T_{dmin} = 1 \text{ Nm}; P_{max} = 30 \text{ N}; T_{dmax}(0) = 4.8 \text{ Nm};$$

$$K(v) = 6.0527 - 0.0607v + 0.0003v^2 - 0.00001v^3.$$

2. Multi-Linear type:

$$T_{dmin} = 1 \text{ Nm}; T_{d1} = 3.5 \text{ Nm}; P_{max} = 30 \text{ N}; T_{dmax}(0) = 4.8 \text{ Nm};$$

$$K(v) = 6.0527 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$

$$K_1(v) = 2.5 - 0.0607v + 0.0003v^2 - 0.00001v^3.$$

3. Nonlinear type:

- Comfortable driving style and sporty driving style:

$$P_{max} = 30 \text{ N}; T_{dmax}(0) = 4.8 \text{ Nm}; K_T = 3.8554;$$

$$K(v) = 6.0527 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$

$$K_a: [b, c, k, n] = [0.2675, 2.7646, 0.4442, 4.5357].$$

- Multi-level driving style:

$$P_{max} = 30 \text{ N}; T_{dmax}(0) = 4.8 \text{ Nm}; K_T = 3.8554;$$

$$T_{drq} = 3.5 \text{ Nm}; v_{rq} = 110 \text{ km/h}; T_{alm} = 2.5 \text{ Nm}$$

$$K(v) = 6.0527 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$

$$K_2(v) = 2.5(110/3.6)/v - 0.0606v + 0.0003v^2 - 0.00001v^3.$$

$$K_a: [b, c, k, n] = [0.2675, 2.7646, 0.4442, 4.5357].$$

4. Mixed type:

- Comfortable driving style and sporty driving style:

$$P_{max} = 30 \text{ N}; T_{dmax}(0) = 4.8 \text{ Nm}; K_T = 3.8554;$$

$$K(v) = 2.5 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$

$$K_1(v) = 6.0527 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$

$$K_a: [b, c, k, n] = [0.2675, 2.7646, 0.4442, 4.5357].$$

- Multi-level driving style:

$$P_{max} = 30 \text{ N}; T_{dmax}(0) = 4.8 \text{ Nm}; K_T = 3.8554;$$

$$T_{drq} = 3.5 \text{ Nm}; v_{rq} = 110 \text{ km/h}; T_{alm} = 2.5 \text{ Nm}$$

$$K(v) = 2.5 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$

$$K1(v) = 6.0527 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$

$$K2(v) = 2.5(110/3.6)/v - 0.0606v + 0.0003v^2 - 0.00001v^3;$$

$$K_a: [b, c, k, n] = [0.2675, 2.7646, 0.4442, 4.5357].$$

3.4. SIMULATION AND EVALUATION OF THE EPS SYSTEM CONTROLLER

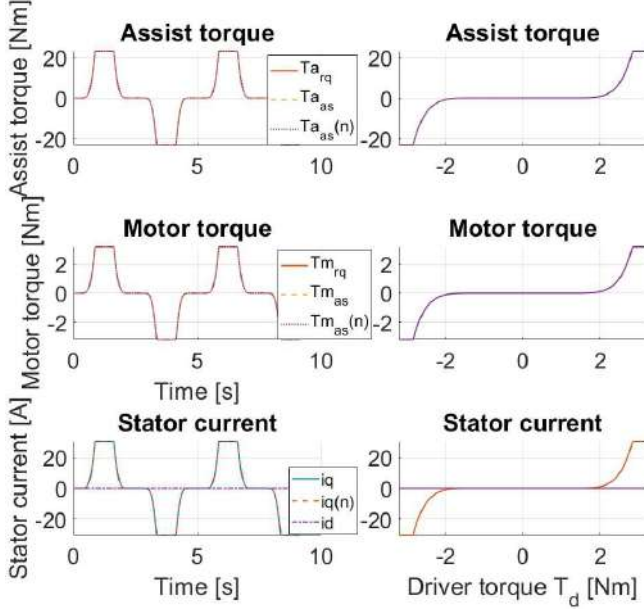


Figure 3.10. Assist torque, motor torque, and motor current

3.5. CHAPTER CONCLUSION

The results of Chapter 3 have demonstrated the validity of the methodology, the reliability of the dynamic models, and the effectiveness of the control strategy. Parameter sets were established for four types of ASC maps, piecewise linear, nonlinear, and mixed, applied to the prototype vehicle. Additionally, parameter sets were defined for two categories (nonlinear and mixed), with three ideal ASC maps developed for each category to reflect three distinct driving styles using the same vehicle configuration.

CHAPTER IV: SIMULATION TESTING AND EVALUATION OF THE EPS

Chapter 4 conducts simulation tests to evaluate the EPS system based on standardized procedures from international ISO and Chinese GB/T standards, aiming to achieve a dual objective: (1) evaluating the controller's response and validating the dynamic models; and (2) assessing the EPS steering feel using the ideal steering assist characteristic maps proposed in the thesis.

4.1. DEVELOPMENT OF THE DYNAMICS AND CONTROL SIMULATION MODEL FOR THE ELECTRIC POWER STEERING SYSTEM

The specifications of the "test vehicle" model and its constituent assemblies are identical to the specifications selected for the study in Chapter 3. The experimental simulation model designated "EPS DYNAMICS AND CONTROL SIMULATION

saturation during high-demand conditions; and (3) adheres to d - q axis principles (generating torque via i_q while maintaining $i_d = 0$). The EPS map block issues appropriate commands, and the integration with CarSim provides the actual load (T_r) to verify robustness. The results demonstrate that the controller responds rapidly and stably under typical excitation conditions for EPS systems.

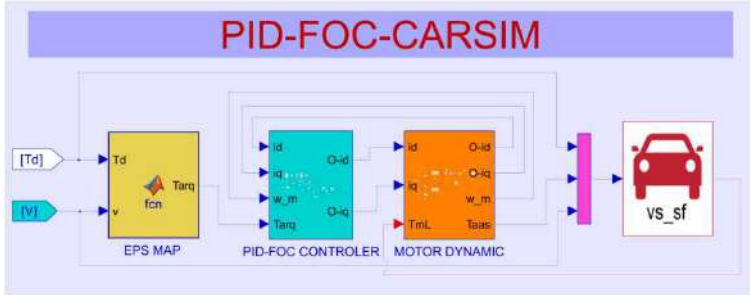


Figure 4.2. Model for evaluating the responsiveness of a PID-FOC controller

4.2.2. Simulation-based evaluation of steering feel

This section employs steering feel tests to achieve two simultaneous objectives: first, to verify dynamic accuracy by comparing the model developed in the Matlab/Simulink environment with the CarSim reference model; and second, to evaluate the steering feel of the EPS system and the suitability of the proposed power-assist characteristic map. Achieving the first objective requires assessing the similarity of dynamic parameters. Achieving the second objective involves evaluating steering feel through objective quantitative metrics, thereby characterizing the overall subjective steering feel and the suitability of the power-assist map. Based on ISO and GB/T standards, the thesis selects test procedures categorized into six main groups that comprehensively reflect the driver's steering feel characteristics:

- 1) On-centre handling test: Evaluates linearity, sensitivity, and stability of the steering system around the center position;
- 2) Pulse input test: Evaluates dynamic response speed and stability under short-duration inputs;
- 3) Returnability test: Evaluates the steering wheel's self-centering capability and trajectory stability after the wheel is released;
- 4) Transition test: Evaluates system responsiveness during changes in motion state;
- 5) Steering effort test at standstill: Evaluates resistive torque and steering feel when vehicle speed is zero or below 5 km/h;
- 6) Step steering input test: Evaluates the system's instantaneous response and overshoot characteristics;

A nonlinear ideal power-assist characteristic map designed for a "comfortable" driving style is utilized. Note: *To distinguish the results between the Simulink and CarSim models, the graphs and values representing the results follow these conventions: (1) "Red" indicates Simulink, and "Blue" indicates CarSim; (2) symbols with the suffix "s" denote Simulink, while those with the suffix "c" denote CarSim.*

4.2.2.1. On-center handling test

Analysis of the metrics for evaluating on-center steering feel, as Figs 4.3 and 4.4.

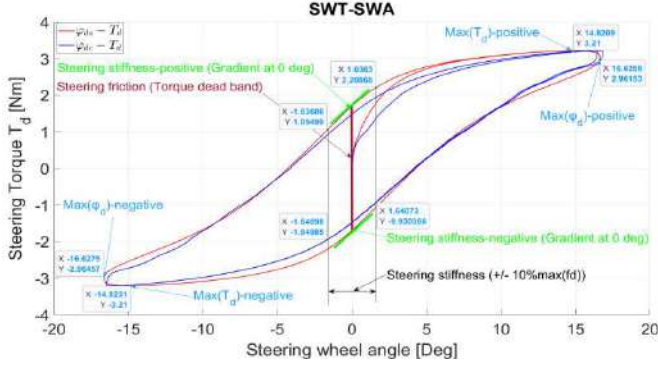
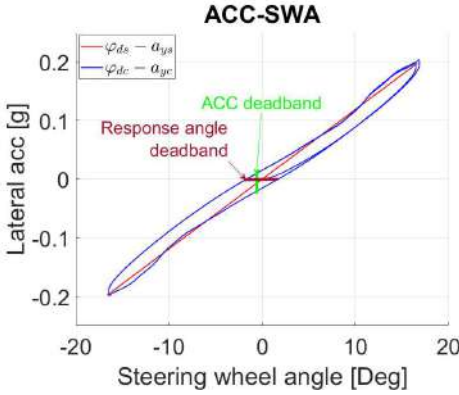
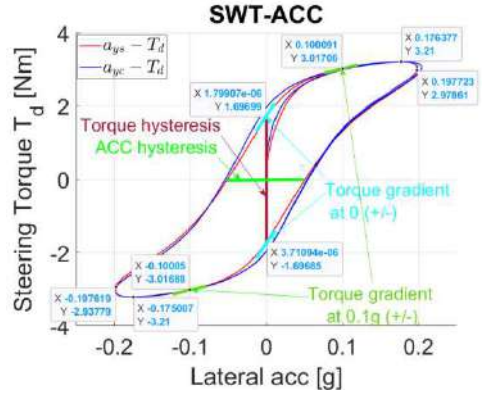


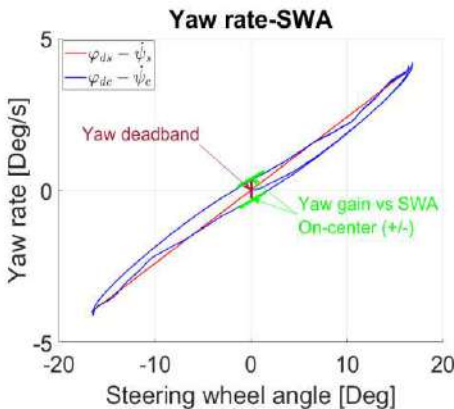
Figure 4.3. Analysis of on-center handling feel, $T_d - \phi_d$ characteristics



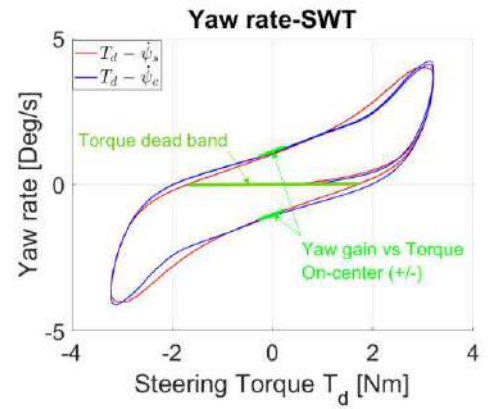
a. $a_y - \phi_d$ characteristics



b. $T_d - a_y$ characteristics



c. $\psi - \phi_d$ characteristics



d. $\psi - T_d$ characteristics

Figure 4.4. Analysis of on-center driving feel and other characteristics.

Evaluation of steering feel and the suitability of the power steering characteristic map: average steering stiffness $(0,1051 \div 0,1123) [Nm/^\circ]$, on-center steering stiffness $(0,3322 \div 0,3646) [Nm/^\circ]$, system friction torque $(1,5752 \div 1,6969) [Nm]$, hysteresis loop area $(58,817 \div 61,6654) [Nm \cdot ^\circ]$, on-center steering angle dead zone $3,3996 [^\circ]$, steering torque gradient at zero acceleration $(28,5964 \div 30,5264) [Nm/g]$, steering torque gradient at $0,1g$ acceleration $(16,9517 \div 17,8450) [Nm/g]$, on-center yaw rate sensitivity to steering angle $(0,2198 \div 0,2405) [(^\circ/s)/^\circ]$, yaw rate response lag to steering angle $0,0570 [s]$, on-center yaw rate sensitivity to steering torque $(0,6003 \div 0,6369) [(^\circ/s)/^\circ]$, and lag time between yaw rate and steering torque $(0,3394 \div 0,3450) [s]$. Based on the steering feel evaluation criteria, the vehicle exhibits a smooth, linear feel with high directional stability; the power steering characteristic map suits a relaxed driving style, although there is a slight friction torque threshold ($\sim 1,6 Nm$) causing a minor delay in response during very small directional reversals.

4.2.2.2. Pulse input test

Under pulse steering input, the power steering characteristic map yields a stable response at all three speeds. At $10 km/h$, $A_0 = 0,061 s^{-1}$ results in smooth steering, though there is residual resonance $f_0 = 2,930 Hz$, $D = +5,303 dB$, $A_p = 0,112$. The response is cleanest at $50 km/h$: $A_0 = 0,216 s^{-1} \approx A_p = 0,215 s^{-1}$, $f_0 = 1,221 Hz$ and $D \approx -0,030 dB$. At $100 km/h$, the vehicle remains stable $A_0 = 0,216$, $f_0 = 1,221 Hz$ và $\gamma^2 = 0,998 - 0,857$ and a phase shift of $13,9 - 64,7^\circ$ across the $0,5 - 1,5 Hz$ frequency range, facilitating predictable steering feel.

4.2.2.3. Returnability test

The self-centering test demonstrates that the steering system possesses good self-centering capability and stability. At the moment of release, $\varphi_0 = 16,247^\circ$ and $\dot{\psi}_0 = 4,053/s$; the response subsequently decays rapidly with $t_r = 2,245 s$, meeting the stability criterion of $\dot{\psi}_s = 0,400^\circ/s$ with zero overshoot ($\sigma = 0\%$). The residual amplitude $\Delta\dot{\psi}$ after 3 seconds is $0,361^\circ/s$; $\dot{\psi}_{max} = 0,761^\circ/s$. The parameters $f_0 = 1,916 Hz$, $D = 0,863$ and $\xi = 0,301$ confirm a smooth steering feel and indicate that the power steering characteristic map meets the required specifications.

4.2.2.4. Transition test

Testing for transient steering feel evaluation: Average steering stiffness $(0,144 \div 0,159) [Nm/^\circ]$, on-center steering stiffness $(1,088 \div 1,200) [Nm/^\circ]$, average lateral acceleration sensitivity $(0,011 \div 0,012) [g/^\circ]$, on-center lateral acceleration sensitivity $(0,011 \div 0,012) [g/^\circ]$, steering torque gradient at zero acceleration $(33,223 \div 35,894) [Nm/g]$, steering torque gradient at $0,1g$ acceleration $(7,658 \div 8,310) [Nm/g]$, on-center yaw rate sensitivity to steering angle $(0,219 \div 0,241) [(^\circ/s)/^\circ]$, and on-center yaw rate sensitivity to steering torque $(0,28 \div 0,031) [(^\circ/s)/^\circ]$. Based on the steering feel evaluation criteria, the steering is smooth with minimal oscillation and safe response, leaning more towards "smooth and solid" characteristics than "sharp and sensitive" ones.

4.2.2.5. Steering effort test at standstill

Average steering stiffness: $(0,0016 \div 0,0018) [Nm/^{\circ}]$; steering stiffness at zero steering angle: $(0,0014 \div 0,0015) [Nm/^{\circ}]$; maximum steering torque: $2.5 [Nm]$. Based on the steering feel evaluation criteria, the steering action is consistent, the effort required is moderate, and the end-stop is distinct; these characteristics are suitable for parking or U-turn maneuvers.

4.2.2.6. Step steering input test

Yaw angle overshoot is $2,484 [\%]$, total variance of yaw rate is $0,084 [s]$, total variance of lateral acceleration is $0,089 [\%]$, yaw rate sensitivity is $0,256 [(^{\circ}/s)/^{\circ}]$, lateral acceleration sensitivity is $0,0124 [g/^{\circ}]$, and the phase lag between steering angle and yaw rate is $0,109 [s]$. Based on the driving feel evaluation criteria, the vehicle exhibits a quick response, good damping, no overshoot, and maintains trajectory stability.

4.3. CHAPTER CONCLUSION

Simulation tests conducted on the prototype vehicle included:

1. Control system testing: The PID-FOC controller demonstrated precise torque tracking capabilities, with near-zero steady-state error, low overshoot, and a rapid response time.

2. Vehicle dynamics verification: Quantitative results showed a difference of less than 10% between the two models, and the waveforms closely matched, confirming a high degree of similarity. This validates that the model was constructed with appropriate structures and parameters, offering sufficient accuracy for application during the design, optimization, and evaluation phases of the EPS system.

Evaluation of steering feel and the suitability of the power steering characteristic map: Overall, the steering feel met the criteria for smoothness, stability, and predictability, aligning with the objectives for an E-segment passenger car that prioritizes comfort over sportiness, while demonstrating strong consistency between subjective perception and quantitative simulation results. The ideal ASC map proved highly optimized, supporting "smooth and stable" operation with precise feedback. This confirms the suitability of the ideal ASC map for the comfort-oriented driving style adopted for the prototype vehicle.

CONCLUSIONS

1. Research results and novel contributions

Research into developing power steering characteristic maps, designing controllers for passenger car Electric Power Steering (EPS) systems, and evaluating steering feel represents an area lacking in-depth domestic study. The dissertation titled "Research on the characteristics and control of passenger car electric power steering systems" constitutes a novel domestic contribution to the field of automotive control dynamics. The dissertation's results and novel contributions include:

The development of ASC maps, there are four novel contributions:

- 1) Establishing a comprehensive process spanning from the development of power steering characteristic maps and controller design to final evaluation via steering feel assessment in accordance with ISO and GB/T standards.

2) Proposing a power steering characteristic map structure comprising three functional modules: "Driving Style," "Vehicle Inverse Dynamics," and "Steering System Inverse Dynamics." This enables the calibration of steering feel and the design of various driving styles on the same vehicle configuration without altering hardware or control logic, thereby saving product development time and reducing production costs. Additionally, the structure offers reusability across various vehicle models and segments.

3) Presenting calculation methods for four distinct types of power steering characteristic maps.

4) Developing an ideal power steering characteristic map aligned with Stevens' power law and designing power steering characteristics based on driving styles constitute a novel contribution.

The development of dynamic models, there are two novel contributions:

1) Proposing a relatively comprehensive formula for determining road resistance torque.

2) Constructing a complete dynamic model of the C-EPS system extending from the steering wheel to the steered wheels, incorporating a realistic Ackermann steering linkage dynamic model.

Regarding controller design, there is one novel contribution: the PID-FOC control strategy for the EPS system represents a new combined control method not previously implemented in domestic research.

Regarding the assessment of steering feel, there is one novel contribution: the thesis introduces a set of criteria and a simulation-based testing method for evaluating steering feel.

The simulation results obtained from application to the prototype vehicle:

1) An integrated simulation model combining Matlab/Simulink and CarSim virtual testing software was developed; this model is capable of performing calculations and simulations for various vehicle segments and establishes a closed-loop process ranging from the construction of steering assist characteristic maps to the quality assessment of these maps via steering feel evaluation.

2) Simulations were conducted to validate the dynamic models using the prototype vehicle.

3) Simulations, analyses, and evaluations were performed for four types of steering assist characteristics: linear, multi-linear, non-linear, and hybrid. Ideal steering assist characteristics were evaluated based on the driving styles associated with non-linear and hybrid types. The resulting parameters for constructing the steering assist characteristic map for the prototype vehicle are as follows:

1. Linear type:

$$T_{dmin} = 1 \text{ Nm}; P_{max} = 30 \text{ N}; T_{dmax}(0) = 4.8 \text{ Nm};$$

$$K(v) = 6.0527 - 0.0607v + 0.0003v^2 - 0.00001v^3.$$

2. Multi-Linear type:

$$T_{dmin} = 1 \text{ Nm}; T_{d1} = 3.5 \text{ Nm}; P_{max} = 30 \text{ N}; T_{dmax}(0) = 4.8 \text{ Nm};$$

$$K(v) = 6.0527 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$

$$K_1(v) = 2.5 - 0.0607v + 0.0003v^2 - 0.00001v^3.$$

3. Nonlinear type:

- Comfortable driving style and sporty driving style:

$$P_{max} = 30N; T_{dmax}(0) = 4.8Nm; K_T = 3.8554;$$
$$K(v) = 6.0527 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$
$$K_a: [b, c, k, n] = [0.2675, 2.7646, 0.4442, 4.5357].$$

- Multi-level driving style:

$$P_{max} = 30N; T_{dmax}(0) = 4.8Nm; K_T = 3.8554;$$
$$T_{drq} = 3.5Nm; v_{rq} = 110km/h; T_{alm} = 2.5Nm$$
$$K(v) = 6.0527 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$
$$K2(v) = 2.5(110/3.6)/v - 0.0606v + 0.0003v^2 - 0.00001v^3.$$
$$K_a: [b, c, k, n] = [0.2675, 2.7646, 0.4442, 4.5357].$$

4. Mixed type:

- Comfortable driving style and sporty driving style:

$$P_{max} = 30N; T_{dmax}(0) = 4.8Nm; K_T = 3.8554;$$
$$K(v) = 2.5 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$
$$K1(v) = 6.0527 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$
$$K_a: [b, c, k, n] = [0.2675, 2.7646, 0.4442, 4.5357].$$

- Multi-level driving style:

$$P_{max} = 30N; T_{dmax}(0) = 4.8Nm; K_T = 3.8554;$$
$$T_{drq} = 3.5Nm; v_{rq} = 110km/h; T_{alm} = 2.5Nm$$
$$K(v) = 2.5 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$
$$K1(v) = 6.0527 - 0.0607v + 0.0003v^2 - 0.00001v^3;$$
$$K2(v) = 2.5(110/3.6)/v - 0.0606v + 0.0003v^2 - 0.00001v^3;$$
$$K_a: [b, c, k, n] = [0.2675, 2.7646, 0.4442, 4.5357].$$

2. Limitations, challenges, and directions for future research

Some challenges associated with the proposed method that require further development include:

- (1) In the proposed method, the parameter (n) may vary with changing driving conditions, across different vehicle types, or according to driver preferences, thereby compromising the model's accuracy in broad applications. (2) Although the method accounts for the effects of speed and vehicle configuration parameters $K_a, K_T, K(v)$, the model does not yet fully reflect variations in road surface conditions, vehicle load, or discrepancies in driver perception. If empirical studies reveal that (n) changes significantly depending on the operating scenario, the argument that no single function can describe perception becomes more compelling. Consequently, future research should focus on refining the model by developing multiple power-steering characteristic maps tailored to specific speed ranges, while also integrating dynamic load factors and variations in the adhesion coefficient to enhance practical applicability.

- Furthermore, the current research findings are limited to simulations and simulation-based testing; designing and conducting real-world experiments to validate the effectiveness of the proposed method represents a necessary direction for future research.

PUBLISHED

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